1. Consider the problem DISJOINTNESS of testing whether two sets $A = \{a_1, \ldots, a_n\}$ and $B = \{b_1, \ldots, b_n\}$ of real numbers are disjoint.

   Outline a simple algorithm for solving this problem for which you can show that it has optimal asymptotic worst case running time.

   What is the algorithm and its running time? For which model of computation do you prove worst case optimality? What is your "lower bound proof?"

2. Consider the problem DIAMETER: For a set $P = \{(u_i, v_i) \,|\, 1 \leq i \leq n\}$ you are to decide whether it contains two points that have distance at least 2 between them.

   Here is an argument that seems to show that DIAMETER is at least as difficult as DISJOINTNESS: Given an instance A, B of DISJOINTNESS we transform it in linear time to an instance P_{AB} of DIAMETER so that the answer to the latter problem is YES iff the answer to the original problem is NO. I.e., P_{AB} contains two points that are at least distance 2 apart iff the sets A and B are not disjoint. Here is how this transformation works: W.l.o.g. assume all reals in A and in B lie in the open interval $(-1, 1)$. (What would you do to enforce this assumption?) Now let

   $$P_{AB} = \{(\cos a_i, \sin a_i)|1 \leq i \leq n\} \cup \{(\cos(\pi + b_j), \sin(\pi + b_j)|1 \leq j \leq n\}\,.$$

   In effect this places the points of A on the "right half" of the unit circle and B on the "left half" in such a way that if A and B have a number in common the corresponging points will be diametrically opposed on the unit circle and the distance between them will be 2.

   a) This argument is fallacious in that it cannot be used to deduce a lower bound for DIAMETER in the algebraic computation tree model. What is wrong with it?

   b) Fix the argument. (Hint: "rational parametrization")