1. Assume there is a box containing n gems of distinct values, with a price tag on each gem telling its value. You are to move the gems from this box to a drop box, one gem after another. You are allowed to keep one of the gems, and of couse you are interested in getting the one of most value. As soon as you have decided to keep a gem, you cannot change this decision any more. At any point in time only one gem is allowed outside the boxes (except for the one that you are allowed to keep). As soon as gem is in the drop box you cannot access it any more.

   Here is a possible strategy: Fix a number $k < n$. Transfer the first k gems and remember the largest value M that you saw. After that transfer the remaining gems and keep the first that you see whose value is bigger than M. (If you don't see such a gem – tough luck.)

   a) What is the probability that you end up with the most valuable gem?
   b) How would you choose k to maximize this probability?

   Here it is assumed that whenever you pick a gem from the box every one of the gems is picked with equal probability. In other words, the order in which you pick all the gems is a random permutation, each with the same probability.

2. A vertical line is drawn on a wall. You want to throw a dart so that it lands as close to the line as possible. You are not a perfect thrower but your throws achieve some continuous distribution of the distance to the line. However, you are a steady thrower in the sense that this distribution is the same with every throw. Moreover, all the throws are independent.

   What is the expected number of throws that you have to make until you get closer to the line than with the first throw?