Algorithms and Data Structures
Winter Term 2015/16
Exercises for Unit 10

1. Let $U = \{0, 1, \ldots, K-1\}$, let $p \geq K$ be a prime number, and let $0 < t < K$. For $0 \leq a, b < p$ define

$$h_{a,b}(x) = (a \cdot x + b \bmod p) \bmod t\,.$$

   Show that the family $\mathcal{H} = \{h_{a,b} \mid 0 < a < p, 0 \leq b < p\}$ is a universal set of hash functions from U to $T = \{0, \ldots, t-1\}$.

2. Let f and g be functions of type $U \to T$. We define (f, g) to be a function of type $U \to T \times T$ specified by $(f, g)(x) = (f(x), g(x))$.

   For a set $\mathcal{H}$ of functions of type $U \to T$ define $\mathcal{H} \star \mathcal{H}$ to be the set of functions $\{(f, g) \mid f, g \in \mathcal{H}\}$.

   Prove: If $\mathcal{H}$ is a universal set of hash functions from U to T, then $\mathcal{H} \star \mathcal{H}$ is a universal set of hash functions from U to $T \times T$.

3. One conceivable way of dealing with collisions in hashing would be to have a whole sequence of hash tables $T_1, T_2, \ldots, T_\ell$, each with its own hash function. If upon inserting item x in T_1 you encounter a collision, you try to insert x in T_2, if there also is a collision there, insert x in T_3, and so on.

   Why is this not a good idea?