Algorithms and Data Structures
Winter Term 2015/16
Exercises for Unit 11

1. For this question assume we have n buckets, $B_1, \ldots, B_n$, initially empty, and an unlimited supply of balls. We repeatedly throw a ball at the buckets and each bucket has the same chance of receiving that ball. All throws are independent of each other.

   a) Prove that after $\Theta(\sqrt{n}))$ balls were thrown, with probability at least 1/2 one of the buckets conmtains at least two balls. What is the best constant that you can prove?

   b) What is the expected number of empty buckets after throwing m balls.

   c) How many balls do you need to throw, so that with probability at least 1/2 every bucket contains some ball?

   d) Prove that after throwing n balls with probability at least $1 - 1/n$ each bucket contains at most $(O((\log n)/\log\log n)$ many balls.

2. Consider the following two functions h_1 and h_2 defined in the following table:

	a	b	c	d	e	f	g
h_1	0	2	2	3	3	0	0
h_2	1	1	2	2	3	3	2

   For each of the sets S_i given below determine the number of ways in which S_i could be stored in the process of cuckoo hashing using two tables, each of size 4, and using the two hash functions h_1 and h_3. Justify your answers.

$$S_1 = \{a, c, e\} \quad S_2 = \{a, b, c, d\} \quad S_3 = \{a, b, c, d, e, f\} \quad S_4 = \{a, b, c, d, e, f, g\}$$