1. Construct an example of a graph with $2k+2$ vertices, $O(k)$ egdes, of which k are in a current matching, so that an appropriate application of Edmond's algorithm for finding an augmenting path involves k blossom shrinking steps.

2. An *odd set cover (OSC)* of an undirected graph $G = (V, E)$ is a family $\mathcal{A}$ of subsets of V so that each $A \in \mathcal{A}$ has odd cardinality, and $\mathcal{A}$ covers all edges of G in the following sense:
   - If $A \in \mathcal{A}$ has cardinality 1, say $A = \{a\}$, then A has "capacity" $c(A) = 1$ and A "covers" all edges incident to a.
   - If $A \in \mathcal{A}$ has cardinality $2k + 1$ with $k \geq 1$, then A has "capacity" $c(A) = k$ and A "covers" all edges in E with both endpoints in A.
   - Each edge of G must be covered.

   Define the capacity of OSC $\mathcal{A}$ as $c(\mathcal{A}) = \sum_{A \in \mathcal{A}} c(A)$. Prove the following theorem due to Jack Edmonds:

   Theorem 1 *In every undirected graph G we have*

   $$\max\{\, |M| \,:\, M \text{ is a matching of } G \,\} = \min\{\, c(\mathcal{A}) \,:\, \mathcal{A} \text{ is an OSC of } G \,\}$$

   Hint: Inequality should be easy to prove. For showing equality, construct an OSC based on the final forest of the (failed) final augmentation attempt in Edmond's algorithm. Blossoms (or superblossoms) always include an odd number of nodes, at least 3, of the original graph...