Algorithms and Data Structures
Winter Term 2015/16
Exercises for Unit 32

1. Let A and B be two multisets containing integers from the range 1 through K. We want to test, whether A and B are the same, i.e. they contain the same elements and with the same respective multiplicities. Of course such an equality test could be performed using order comparisons – essentially via sorting. It could be also solved via universal hashing.

   Design a method for solving this problem via polynomial identity testing. What kind of runing time and probabilistic guarantees can you achieve?

2. Let $P(x)$ be a polynomial over the reals that is not the zero polynomial. It is well known that the number of positive roots of P is at most the number of non-zero coefficients of P minus one (a consequence of the sign rule of Descartes). Use this fact to prove the following "Schwartz-Zippel"-like lemma:

   Lemma 1 *Let U be a set of L positive integers. Let $P(x_1, \ldots, x_n)$ be a multivariate polynomial that consists of $k + 1$ monomials (and is therefore not the zero polynomial).*

   If values for the x_i's are chosen from U independently and uniformaly at random, then $P(x_1, \ldots, x_n) = 0$ with probability at most k/L.

3. (only if you have extra time)

   Let G be a bipartite graph, and let A be its Edmonds matrix. Prove the following:

   The size of the maximum matching of G equals the rank of A.