1. Show that the price of anarchy for routing games with quadratic latency functions $\ell_e(x) = x^2$ is at most
$$\frac{1}{1 - \frac{2}{3\cdot\sqrt{3}}} \approx 1.625 \ .$$

2. For a non-atomic routing game with affine latency functions $\ell_e(x) = a_e x + b_e$ consider the following convex function
$$\Phi(f) = \sum_{e\in E} \frac{a_e}{2} f_e^2 + b_e f_e \ .$$
Show that f is a Wardrop equilibrium if and only if it is a feasible flow that minimizes Φ. How would you define a similar function Φ for routing games with more general latency functions?

3. The government decided to collect a route tax of $\tau_e(x)$ to improve the overall congestion in the traffic network. Suppose for every edge $e \in E$ they use
$$\tau_e(x) = x \cdot \frac{d}{dx}\ell_e(x) \ ,$$
the *marginal cost* of edge e. Thus, the taxed latency of edge e is $\ell_e^\tau(x) = \ell_e(x) + \tau_e(x)$.

   Suppose ℓ_e are affine latency functions and let f^* be an optimal flow. Show that f^* is an equilibrium flow w.r.t. taxed latency ℓ^τ. Does this property also hold for routing games with more general latency functions?