1. For stable matching with incomplete lists, each man $x \in \mathcal{X}$ has a strict list $\succ_x$ over a subset of the women $\mathcal{Y}$, i.e., $\succ_x$ is possibly incomplete. Similarly, a woman $y \in \mathcal{Y}$ might not rank all men in $\mathcal{X}$. Pairs (x, y) where x does not rank y or y does not rank x are *forbidden*.

   Equivalently, we may think of this scenario as $\succ_x$ being a complete strict preference list over $\mathcal{Y} \cup \{x\}$. If $x \succ_x y$, this means x prefers to stay alone rather than be matched to y. Similarly, each $y \in \mathcal{Y}$ has a list $\succ_y$ over $\mathcal{X} \cup \{y\}$.

   Using this formulation, the definitions of blocking pair and stable matching can directly be generalized. Show that there is always a stable matching and how to compute it efficiently, even when we have incomplete lists.

2. With incomplete lists, some agents may remain single in a stable matching. We show that all stable matchings match the same set of agents. More formally, consider any instance of stable matching with incomplete lists and show the following statement:

   For two stable matchings M and M', man x is matched in M if and only if x is matched in M'. Similarly, woman y is matched in M if and only if y is matched in M'.

3. Consider an instance of stable matching with incomplete lists, and let M^* be a maximum bipartite matching of non-forbidden pairs. Show that the price of anarchy for stable matching is 2: Every stable matching has at least half the number of pairs of a maximum bipartite matching, $|M| \geq |M^*|/2$.

4. Suppose the preference lists of one side, say the woman-side, are private information of the agents. Show that the DA algorithm is not a truthful mechanism for women. More precisely, give an example of a set of preference lists, where for woman y we have $x \succ_y x'$, but both men x and x' are low on y's list. Now instead of the real list $\succ_y$, in the beginning y reports a false list $\succ'_y$, where she switches the order of x and x' such that running the DA algorithm with $\succ'_y$ assigns her to a better man x'' with $x'' \succ_y x$.

5. Consider the dynamics studied in class where we iteratively resolve blocking pairs. Give an instance with an initial matching and a sequence of blocking pair resolutions such that the sequence runs into a cycle.

6. Stable roommates is the non-bipartite version of stable matching. In stable roommates we have a set N of n agents, and each agent i has a preference list $\succ_i$ over $N \setminus \{i\}$. A blocking pair is defined as before (no pair of agents can strictly improve by matching up), and a stable matching again is a matching without blocking pair.

   a) Construct a stable roommates instance without a stable matching.

   b) Does a stable matching exist in correlated stable roommates instances, where preference lists are based on edge weights?