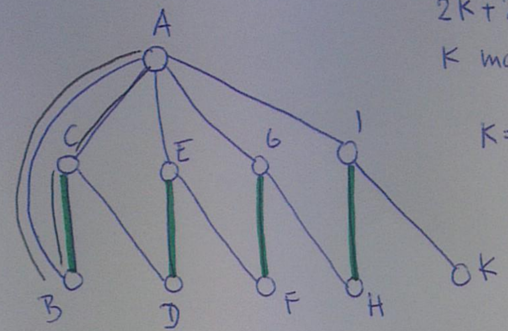
Exercises for Unit 31

(1.)

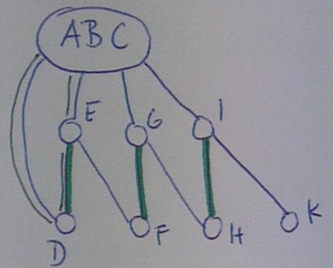


$2K+2$ vertices, $O(K)$ edges and K matched edges

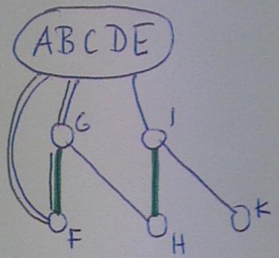
$K=4$, can make the example as large as we want to

— → how we traverse the graph

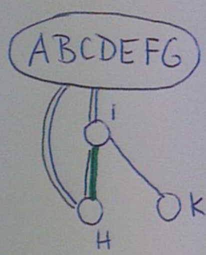
1. ⇒



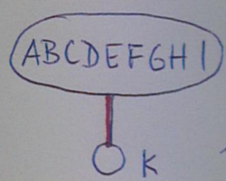
2. ⇒



3. ⇒



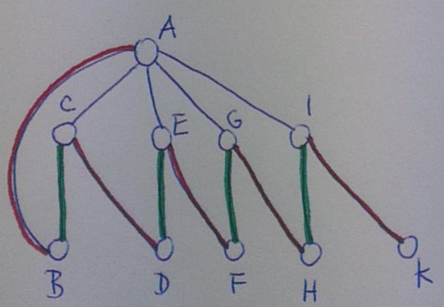
4. ⇒



→ both free

⇒ augmenting path

– the augm. path in the original graph:



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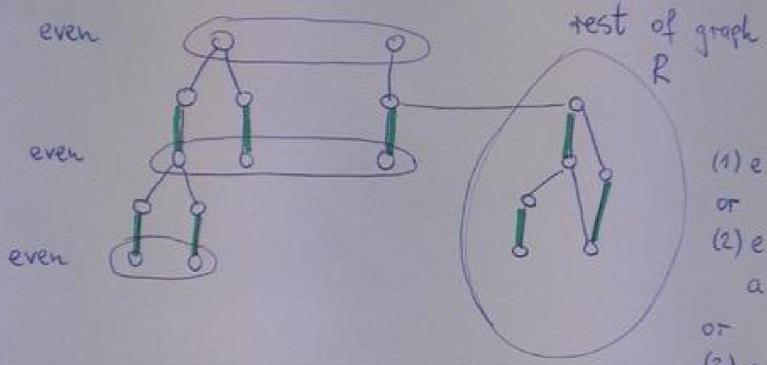
(2.) $\boxed{\leq}$ any matching M can only contain $c(V_i)$ edges from the edges covered by $A \in \mathcal{A}$

$\Rightarrow |M| \leq c(\mathcal{A})$

$\boxed{\geq}$ consider the situation at termination of Edmonds's algorithm.

M_0' matching in G', M_0 "expanded" matching in G



every edge $e \in E(G)$:

(1) e is incident to an odd node

or

(2) e is contained in a (contracted) blossom of G'

or

(3) e is contained in R

- define OSC γ_0:

covers all edges

(a) all odd nodes put as singletons (contracted nodes are not odd)

(b) contracted nodes $\rightarrow$ non-singleton sets V_i

note: $c(V_i) = \#$ matching edges in V_i

(c) $k := \#$ matching edges in $R \Rightarrow R$ has $2k$ nodes (because the rest of the graph is fully matched)

- $k = 0 \Rightarrow \gamma_0$ defined by (a) and (b) and $|M_0| = c(\gamma_0)$
- $k = 1 \Rightarrow$ add a singleton V_j with one node from R to γ_0
- $k \geq 2 \Rightarrow$ add one singleton V_j with one node from R and put all other $2k-1$ nodes into V_k

$\Rightarrow c(V_j) + c(V_k) = 1 + k - 1 = k$

all cases $|M_0| = c(\gamma_0)$