Algorithms and Data Structures
Spring 2018
Exercises for Units 27 and 28

1. Let S be a set of n segments in the plane, each of which is either horizontal, i.e. it is spanned by two points $(a_1, b), (a_2, b)$, or vertical, i.e. it is spanned by two points $(a, b_1), (a, b_2)$.

   Develop a plane sweep algorithm that determines whether any two of the segments in S intersect.

   Be specific about invariants, events, event queue and sweep line structure.

   What running time does your algorithm achieve? Ideally it should be $O(n \log n)$.

2. Assume the same setting as in question 1. But now your algorithm is to count the number of intersecting segment pairs.

   Again, what are your invariants and what structures do you use?

   Ideally you should again achieve a running time of $O(n \log n)$

   You may make the simplifying assumption that neither any two horizontal segements intersect each nor do any two vertical segments.

3. Again you are given a set S of n segments all of which are eigher horizontal or vertical. The distance between two segments s and t is defined as $d(s,t) = \min\{d(u,v) | u \in s,\ v \in t\}$.

   Develop an algorithm that determines the pair of distinct segments in S that have the smallest distance to each other.

   (Hint: It may be useful to make case distinction as to how the distance between two segments is realized, i.e. how many endpoints are involved.)

4. The diameter $\Delta(S)$ of a finite set S is defined as $\Delta(S) = \max\{d(u,v) | u, v \in S\}$.

   (a) Prove that for any pair p, q of points in a planar set S that realize the diameter of S it must be the case that both p and q must be extreme points of S, i.e. vertices of the convex hull of S.

   (b) Give an algorithm that given the convex hull of a finite set S of points in the plane computes $\Delta(S)$ in linear time.

5. The *width* of a set S in the plane is defined to be the smallest distance between two parallel lines in the plane that have S in slab between them (the lines count as part of the slab).

   Develop an algorithm that given a set S of n points in the plane computes the width of S.

6. Let P and Q be two convex polygons in the plane, each specified by the circular sequence of its vertices.

   (a) What is the maximal number of intersections that can occur between the boundaries of P and Q, if P has m vertices and Q has n vertices?

   (b) Develop an algorithm for computing the convex hull of $P \cup Q$. Ideally you should achieve linear running time.

   Hint: Although no doubt a complicated, specialized algorithm for this problem can be formulated, there is really no need to do so in order to solve this problem.