1. Let B be a set of $2b+1$ blue points in the plane. A *halving line for* B is a line that contains one point of B and has at most b points of B on each side.

   (a) Apply duality (say the one of the form $y+d=kx \leftrightarrow (k,d)$) and characterize in the dual the set of all halving lines.

   Now let R be a set of $2r+1$ red points.

   The so-called "Ham-Sandwich Theorem" states that there must be a line that is simultaneously a halving line for B and for R.

   (b) Use your characterization to develop a proof of the Ham-Sandwich Theorem.

2. Let S be a set of n sites in the plane and let T be a triangulation for S.

   Call an edge e of T *locally Delaunay* if either e is part of just one triangle of T, or otherwiese in the case that e is part of two triangles t_1 and t_2, if the open circumdisk of t_1 contains no vertex of t_2 in its interior and likewise the open circumdisk of t_2 contains no vertex of t_1 in its interior.

   Obviously in the Delaunay triangulation all edges are locally Delaunay. It turns out that the converse also holds, i.e. if all edges of a triangulation T are locally Delaunay, then T must be the Delaunay triangulation.

   This characterization allows to check in linear time whether a given triangulation of S is the Delaunay thriangulation. It also gives rise to natural algorithms to compute the Delaunay triangulation:

   In a triangulation T call an edge e with incident triangles t_1 and t_2 *eligible* if the union of t_1 and t_2 is a convex quadrilateral. Call e *ineligible* otherwise.

   (c) Prove that every ineligible edge of T is locally Delaunay.

   A *flip* of an eligible edge e in a triangulation T changes T to a new triangulation T' by replacing e by the other diagonal e' of the quadrilateral formed by $t_1 \cup t_2$ (and replace t_1 and t_2 accordingly).

   (d) Prove that if an eligible edge e that is not locally Delaunay is flipped, then the new edge e' will be locally Delaunay in the changed triangulation T'.

   Accordingly we call such a flip a *Delaunay flip*.

   (e) Consider the points of S vertically mapped onto the unit paraboloid $z = x^2 + y^2$, and consider triangles of T now spanned by the mapped points in $\mathbb{R}^3$. Try to characterize Delaunay flips in the 3-dimensional view.

   The flip algorithm for producing a Delauny triangulation for S proceeds as follows: At first compute some triangulation of S. Then, as long as there is some eligible edge in the (current) triangulation that is not locally Delaunay, flip that edge.

   Obviously, if this algorithm terminates, it terminates with the Delaunay triangulation. But it is not clear that it actually terminates, since some cyclic flipping behaviour is conceivable.

   (f) Prove that this flip algorithm terminates, and that it does so after fewer than $\binom{n}{2}$ flips.